Lesson Summary

It is not clear how to perform arithmetic on numbers given as infinitely long decimals. If we approximate these numbers by truncating their infinitely long decimal expansions to a finite number of decimal places, then we can perform arithmetic on the approximate values to estimate answers.

Truncating a decimal expansion to n decimal places gives an approximation with an error of less than $\frac{1}{10^n}$. For example, 0.676 is an approximation for $0.676767\ldots$ with an error of less than 0.001.

Problem Set

1. Two irrational numbers x and y have infinite decimal expansions that begin $0.3338117\ldots$ for x and $0.9769112\ldots$ for y.
   a. Explain why 0.33 is an approximation for x with an error of less than one hundredth. Explain why 0.97 is an approximation for y with an error of less than one hundredth.
   b. Using the approximations given in part (a), what is an approximate value for $2x(y+1)$?
   c. Repeat part (b), but use approximations for x and y that have errors less than $\frac{1}{10^6}$.

2. Two real numbers have decimal expansions that begin with the following:

$$x = 0.70588\ldots.$$

$$y = 0.23529\ldots.$$

   a. Using approximations for x and y that are accurate within a measure of $\frac{1}{10^2}$, find approximate values for $x + 1.25y$ and $\frac{x}{y}$.
   b. Using approximations for x and y that are accurate within a measure of $\frac{1}{10^4}$, find approximate values for $x + 1.25y$ and $\frac{x}{y}$.
   c. We now reveal that x and y are rational numbers with the property that each of the values $x + 1.25y$ and $\frac{x}{y}$ is a whole number. Make a guess as to what whole numbers these values are, and use your guesses to find what fractions x and y might be.